A CONCEPTUAL STUDY ON PERFECT NUMBER

An Extra Mural Seminar By Prof Ashok Kumar Das,
HOD, Mathematics , Kendrapara Autonomous College,
Kendrapara

2019-20

Organiged by Department of Mathematics,
Pattamundai College, Pattamundai
On 10th February 2020

REPORT

A seminar was organised by Department of Mathematics, Pattamundai College, Pattamundai on 10.02.2020 on the topic "A CONCEPTUAL STUDY ON PERFECT NUMBER". Prof Ashok Kumar Das , HOD, Mathematics, Kendrapara Autonomous College, Kendrapara who graced the seminar with his analytical thinking. We were able to get the beautiful glimpses of the students of our Department. Sri Arabinda Pandab, Head of the Department gave a key note address of the topic and welcomed the guests on the diace and the participants. The meeting was ended with a vote of thanks by Dr Nirmal Kumar Sahoo, another faculty member.

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A seminar talk on
" A Conceptual Study on Perfect Numbers"

-Prof Ashok Kumar Das, HOD, Mathematics,
Kendrapara Autonomous College,Kendrapara

In number theory, a Perfect number is aPositive integer that is equal to the sum of its positive divisors excluding the number itself. For example '6' has divisors 1,2and 3(excluding itself).

And 1+2+3=6, So "6" is a Perfect number.

The sum of divisors of a number, excluding the number itself is called its aliquot sum. So, a Perfect number is one that is equal to its aliquot sum. Equivalently a Perfect number is a number that is half the sum of all its Positive divisors including itself. For example "28" is a Perfect number as its Positive divisors are 1,2,4,7,14and 28 and

1+2+4+7+14+28=2x28=56

Euclid also proved a formation rule i.e, q (q +1)/2 is an even perfect number whenever q is q prime of the form 2^{p}-1 for prime P-what is now called a Mersenne Prime. Later Euler proved that all even Perfect numbers are of this form. So, this is known as Euclid-Euler theorem.

It is not known whether there are any odd Perfect numbers, nor whether infinitely many Perfect numbers exist. The first few Perfect numbers are 6, 28,496 and 8128.

The sum of the Positive divisors of an integer n each of them less than n is given by

$^{\delta}$ (n)-n i.e., if $^{\delta}$ (n)-n=n then n is Perfect.

So $^{\delta}$ (n) =n+n=2n

Only four Perfect numbers were known to ancient Greeks. Nicomachus says that they formed in an orderly fashion one among units, one among the tens, one among the hundreds and one among the thousands (that is less than 10,000)

i,e; i) The nth Perfect number P_n contains exactly n digits.

ii)The even Perfect numbers end, alternately in 6and 8

But it is wrong because there is no Perfect number with 5 digits. The next Perfect number is P_5 =33550336, although the succeeding perfect number P_6= 8589869056 which also ends in 6, not 8 as alternately. So, we conclude that the even Perfect numbers always end in 6 and 8but not alternately.

The Problem of determining the general form of all Perfect numbers was partially solved by Euclid that is if the sum

$$1+2+2^2+2^3+\text{-----------}+2^{k-1}=p$$

is a Prime number, then $2^{k-1}p$ is a Perfect number i,e; $1+2+2^2=7$.

Hence $2^2 x 7=28$ is a Perfect number.

Similarly, for p=2, $2^1(2^2-1) =2x3=6$

for p=3, $2^2(2^3-1) =4x7=28$

for p=5, $2^4(2^5-1) =16x31=496$

for p=7, $2^6(2^7-1) =64x127=8128$

Prime number s of the form 2^p-1 are known as Mersenne Primes.

Nichomachus discovered that every Perfect number is of the form $2^{p-1}(2^p-1)$ where, 2^p-1 is a prime. Until 18th Century Euler proved that that the formula $2^{p-1}(2^p-1)$ will yield all the even perfect numbers. So, there exists one-to-one correspondence between even perfect numbers and the Mersenne Primes, each Mersenne Primes generate one even perfect number and vice versa.

An exhaustive search shows that 47 perfect numbers are $2^{p-1}(2^p-1)$ for,

P=2,3,5,7,13,17,19, ...,42643801 and 43112609.

For higher perfect numbers value also been discovered, namely those for which

P= 57885161,74207281,77232917 and 82589933. As 7 December 2018, 51 Mersenne primes are known and 51 Perfect numbers. But it is not known whether there are infinitely many Perfect numbers nor whether there are infinitely many Mersenne Primes.

Except 6 other Perfect numbers,

$6=2^1(2^2-1) =1+2+3$

$28=2^2(2^3-1) =1+2+3+4+5+6+7=1^3+3^3$

$496=2^4(2^5-1) =1+2+3+\text{------------}+29+30+31=1^3+3^3+5^3+7^3$

$8128=2^6(2^7-1) =1+2+3+4+5+\text{--------------}+127=1^3+3^3+5^3+7^3+\text{------------}+15^3$

$33550336=2^{12}(2^{13}-1) =1+2+3+4+55+\text{-----------------}+8191$

$=1^3+3^3+\text{-------------------}+125^3+127^3$

can be expressed as sum of the cube of odd numbers.

Due to their form, $2^{p-1}(2^p-1)$, every even Perfect number is represented in binary

form as P ones followed by p-1 zero, examples,

$(6)_{10}=2^2+2^1= (110)_2$

$(28)_{10}=2^4+2^3+2^2= (11100)_2$

$(496)_{10}=2^8+2^7+2^6+2^5+2^4=(111110000)_2$

Similarly,

$(8128)_{10}=2^{12}+2^{11}+2^{10}+$-------------------$+2^6=(1111111000000)_2$

Thus every even Perfect number is a Pernicious number i,e; the sum of the digits of its binary representation is a prime number.

Example: If 2^k-1 is Prime$(k>1)$ then $n=2^{k-1}(2^k-1)$ is perfect i,e, every even Perfect number is of this form.

Proof:

Suppose $2^k-1=p$ is prime.

Consider $n=2^{k-1}.\ p$

So $\gcd(2^{k-1}, p)=1$.

So $^{\delta}(n)={}^{\delta}(2^{k-1}.\ p)$

$={}^{\delta}(2^{k-1})\,{}^{\delta}(p)$

$=(2^k-1).2^k$ (As $2^k-1=p$)

$=2^{k-1}.2(2^k-1)$

$=2.\ 2^{k-1}(2^k-1)$

$=2.\ 2^{k-1}p$

$=2n$

Hence n is a Perfect number.

Example:

An even Perfect number of n ends in the digit 6 or 8, equivalently,

either $n\equiv6 \pmod{10}$

or $n\equiv8 \pmod{10}$.

Proof:

Since n is an even Perfect number, so n be represented as $n=2^{k-1}(2^k-1)$

Where 2^k-1 is a prime. But previously we know K must also be Prime.

If $k=2$, then, $n=6$ and the asserted result holds.

We may give our aim to the case $k>2$.

The proof comes to two parts, i.e; $k=4m+1$ or $k=4m+3$.

If $k=4m+1$,

$n=2^{4m}(2^{4m+1}-1)$

$=2^{8m+1}-2^{4m}$

$=2.16^{2m}-16^m$

So, by induction, $16^t\equiv6(\text{mod}10)$ for any positive integer t using this congruence.

We have $n\equiv2.6-6\equiv6 \pmod{10}$

If $k=4m+3$

$n=2^{4m+2}(2^{4m+3}-1)$

$=2^{8m+5}-2^{4m+2}$

$=2.16^{2m+1}-4.16^{m}$

Using the fact that $16^{t}\equiv 6 \pmod{10}$

$n \equiv 2.6-4.6 \equiv -12 \equiv 8 \pmod{10}$

So, every even perfect number has a last digit equal to 6 or to 8.

Ashok Kumar Das.
10/2/20.

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Ref No. : 258 Date 08/02/2020

To

Prof. Ashok Kumar Das
HOD, Mathematics,
Kendrapara Autonomous College,
Kendrapara.

Sub: - An invitation as Resource Person in the Extramural Seminar in Department of Mathematics.

Sir,

It is my pleasure to invite you as **Resource Person** in the Extramural Seminar on the topic **"A Conceptual Studies on Perfect Number"** to be organiged by Department of Mathematics, at 10.00 am on 10th February 2020 in our institution.

Your kind presence for this occasion is highly solicited.

Yours Faithfully,

8.2.2020

Principal
Pattamundai College,

Principal
Pattamundai College

An Extramural Seminar on "A Conceptual Study on Perfect Number"
Department of Mathematics
Pattamundai College, Pattamundai

Dt. 10.02.2020

Sl No	Name of the Resource Person/Faculty Member	Signature & Mobile Number
1	Prof Ashok Kumar Das	Ashok Kumar Das 9937402881.
2	Sri Arabinda Pandab	[illegible] 10/2/20. 9437254024
3	Dr. Nirmal Kumar Sahoo	Nirmala Kumar Sahoo 8974405754.
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Department of Mathematics
Pattamundai College Pattamundai
Extramural Seminar on "A Conceptual Studies on Perfect Number"
on Dt.10.02.2020

Students Attendance

Sl No	Name of the Student	Roll No	Signature of the Student
1	Swagatika Das.	BS-17-104	Swagatika Das.
2	Manasi Mohanty	BS-17-119	Manasi Mohanty
3	Sagarika Nayak	BS-17-054	Sagarika Nayak
4	Aryabartini Jena	BS(P)19-053	Aryabartini Jena
5	Jyotiprava Sahoo	BS(P)19-036	Jyotiprava Sahoo.
6	Kabita Patra	BS(P)19-077	Kabita patra
7	Manisha Swain	BS-18-139	Manisha Swain
8	Elina Swain	BS-18-138	Elina Swain
9	Usharani Mohanty	BS-18-080	Usharani Mohanty
10	Saswatika Rath	BS-18-128	Saswatika Rath
11	Digantika Das	BS-18-140	Digantika Das
12	Steshree Mahanta	BS-18-064	Steshree Mahanta.
13	Ashrita Bhuyan	BS-18-026	Ashrita Bhuyan
14	Anyasha Panda	BS18-004	Anyasha Panda
15	Mahamayee Sahoo	BS17-159	Mahamayee Sahoo
16	Swatimanjari pradhan.	BS17-065	Swatimanjari pradhan.
17	Rojalin Barik	BS17-145	Rojalin Barik
18	Nandakishore Rout	BS-17-039	Nandakishore Rout
19	Satya Narayan Das	BS-18-091	Satya Narayan Das
20	Jyoti Keshari Satapathy	BS-19-114	Jyoti Keshari Satapathy
21	Subhakanta Rath	BS-19-026	Subhakanta Rath
22	Chandan senapati	BS-19-118	Chandan senapati
23	Sudhir kanta Das	BSP-19-005	Sudhir kanta Das
24	Manoj kanta Sahoo	BSP-19-060	Manoj kanta Sahoo.
25	Goutam Behera	BS(P)-19-065	Goutam Behera
26	Nalinikanta Patra	BS(P)-19-099	Nalinikanta Patra
27	Raghabendra Biswal	BS-18-045	Raghabendra Biswal
28	Kailash ch Malik	BS-18-075	Kailash ch Malik
29	Sai prasad swain	BS-18-100	Sai prasad swain
30	Dipak kumar pani	BS-19-056	Dipak kumar pani
31	Mihir Ranjan Tripathy	BS-19-054	Mihir Ranjan Tripathy

Sl No	Name of the Student	Roll No	Signature of the Student
32	Swaraj kumar Das	BS17-068	Swaraj Ku Das
33	Sudhansu sekhar Rout	BS17-111	Sudhansu sekhar Rout
34	Debabrata unatha	BS 17-158	Debabrata unatha
35	Yamunakanta Roy	BS-18-105	Yamunakanta Roy
36	Malaya Ranjan Samy	BS-18-070	Malaya Ranjan Samy
37	[illegible]	BS-17-074	[illegible]
38	Dibyasundar Behera	BS-17-132	Dibyasundar Behera
39	Ansuman Sahoo	BS-17-135	Ansuman Sahoo.
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A Seminar on
'A Conceptual Studies on Perfect Number'
Resource Person: Prof Ashok Kumar Das
HOD, Mathematics, Kendrapara Auto College
Date: 10.02.2020
Organised By : DEPARTMENT OF MATHEMATICS
PATTAMUNDAI COLLEGE, PATTAMUNDAI


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